Transistor Equations

Circuit gain and impedance characteristics are given in terms of transistor parameters for grounded base, grounded emitter and grounded collector configurations. Simplifying approximations are given where appropriate

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THE ACCOMPANYING tabulation summarizes some of the important circuit equations useful to engineers in the application of transistors.

All equations are given in terms of the transistor parameters: collector resistance r_c, base resistance r_b, emitter resistance r_e and current amplification constant α. These quantities are all described in references listed in the bibliography. The quantity r_c is almost always much larger than r_b and r_e and often is even much larger than the load resistance. This makes possible approximations that greatly simplify the complicated exact equations. To evaluate these approximations in this tabulation, the exact expression is always given first followed, where appropriate, by a simpler approximation equation.

The other quantities listed are self-explanatory.

BIBLIOGRAPHY

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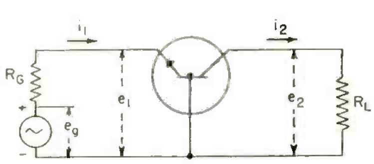
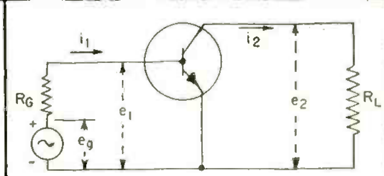
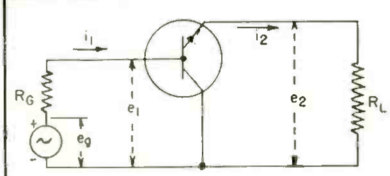
	GROUNDED BASE	GROUNDED EMITTER	GROUNDED COLLECTOR
			
$\frac{e_2}{e_1}$	$= \frac{\alpha R_L}{r_e + r_b(1-\alpha) + \frac{(r_e + r_b) R_L}{(r_c + r_b)}}$ IF $R_L \ll r_c$ $\cong \frac{\alpha R_L}{r_e + r_b(1-\alpha)}$	$= \frac{-R_L\left[\alpha - \frac{r_e + r_b}{r_c + r_b}\right]}{r_e + r_b(1-\alpha) + \frac{(r_e + r_b) R_L}{(r_c + r_b)}}$ IF $R_L \ll r_c$, $r_e + r_b \ll r_c$ $\cong \frac{-\alpha R_L}{r_e + r_b(1-\alpha)}$	$= \frac{r_c}{r_c + r_b}\left[\frac{1}{\frac{R_L + r_e}{R_L} + \frac{r_b(1-\alpha)}{R_L}}\right]$ IF $r_e \ll R_L$, $r_b(1-\alpha) \ll R_L$, $r_b \ll r_c$ $\cong$ UNITY
$\frac{e_2}{e_g}$	$= \frac{\alpha R_L}{\left[r_e + r_b + R_G\right]\left[1 + \frac{R_L}{r_c + r_b}\right] - \alpha r_b}$ IF $R_L \ll r_c$ $\cong \frac{\alpha R_L}{r_e + R_G + r_b(1-\alpha)}$	$= \frac{-R_L\left[\alpha - \frac{r_e + r_b}{r_c + r_b}\right]}{\left[r_e + r_b\right]\left[1 + \frac{R_L}{r_c + r_b}\right] + R_G\left[1 + \frac{R_L + r_e}{r_c + r_b}\right] - \alpha(r_b + R_G)}$ IF $R_L + r_e \ll r_c$, $r_e + r_b \ll r_c$ $\cong \frac{-\alpha R_L}{r_e + (1-\alpha)(r_b + R_G)}$	$= \frac{1}{\left[1 + \frac{r_e}{R_L}\right]\left[1 + \frac{r_b + R_G}{r_c}\right] + \left[\frac{r_b + R_G}{R_L}\right]\left[1 - \alpha\right]\left[1 + \frac{r_b}{r_c}\right]}$ IF $r_e \ll R_L$, $r_b + R_G \ll r_c$, $r_b \ll r_c$ $\cong \frac{1}{1 + \left[\frac{r_b + R_G}{R_L}\right]\left[1 - \alpha\right]}$
$\frac{i_2}{i_1}$	$= \frac{\alpha}{1 + \frac{R_L}{r_c + r_b}}$	$= \frac{-\left[\alpha - \frac{r_e + r_b}{r_c + r_b}\right]}{\left[1 - \alpha\right] + \frac{R_L + r_e}{r_c + r_b}}$ IF $r_e \ll r_c$, $r_e \ll R_L$, $r_b \ll r_c$ $\cong \frac{-\alpha}{(1-\alpha) + \frac{R_L}{r_c}}$	$= \frac{1}{\left[1 - \alpha\right]\left[\frac{r_c + r_b}{r_c}\right] + \frac{R_L + r_e}{r_c}}$ IF $r_b \ll r_c$, $r_e \ll R_L$ $\cong \frac{1}{(1-\alpha) + \frac{R_L}{r_c}}$

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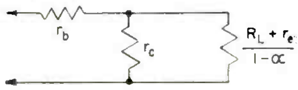
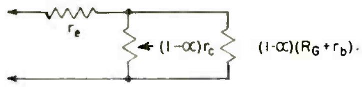
Transistor Equations (continued from p 156)

	GROUNDED BASE	GROUNDED EMITTER	GROUNDED COLLECTOR
INPUT RESISTANCE R_{IN}	$= r_e + r_b\left[1 - \frac{\alpha}{1+\frac{R_L}{r_c+r_b}}\right]$ IF $R_L \ll r_c$ $\cong r_e + r_b(1-\alpha)$	$= r_b + r_e\left[\frac{\frac{r_c+R_L}{r_c+r_b}}{\frac{R_L+r_e}{r_c+r_b}+(1-\alpha)}\right]$ IF $r_b \ll r_c$, $r_e \ll R_L$ $\cong r_b + r_e\left[\frac{1+\frac{R_L}{r_c}}{1-\alpha+\frac{R_L}{r_c}}\right]$ IF IN ADDITION $R_L \ll r_c$ $\cong r_b + \frac{r_e}{1-\alpha}$	$= r_b + \frac{r_c}{1+\frac{(1-\alpha)(r_c+r_b)}{R_L+r_e}}$ IF $r_b \ll r_c$ THIS IS EQUIVALENT TO 
OUTPUT RESISTANCE R_{OUT}	$= (r_c+r_b)\left[\frac{r_e+r_b(1-\alpha)+R_G}{r_e+r_b+R_G}\right]$ IF $r_b \ll r_c$ $\cong r_c\left[\frac{r_e+r_b(1-\alpha)+R_G}{r_e+r_b+R_G}\right]$	$= r_c\left[\frac{\left[1+\frac{r_b+R_G}{r_c}\right]\left[r_e+(1-\alpha)r_b\right]+R_G(1-\alpha)}{r_e+r_b+R_G}\right]$ IF $r_b+R_G \ll r_c$ $\cong r_c\left[\frac{r_e+(1-\alpha)(r_b+R_G)}{r_e+r_b+R_G}\right]$	$= r_e + \frac{(1-\alpha)(r_c+r_b)}{1+\frac{r_c}{R_G+r_b}}$ IF $r_b \ll r_c$ THIS IS EQUIVALENT TO 
OPERATING GAIN = POWER TO LOAD / MAX. POWER FROM GEN.	$= \frac{4R_LR_G\alpha^2}{\left\{\left[\frac{R_L}{r_c+r_b}\right]\left[r_e+r_b+R_G\right]+\left[r_e+R_G+r_b(1-\alpha)\right]\right\}^2}$ IF $R_L \ll r_c$ $\cong \frac{4R_LR_G\alpha^2}{\left[r_e+R_G+r_b(1-\alpha)\right]^2}$	$= \frac{4R_GR_L\left[\alpha-\frac{r_e+r_b}{r_c+r_b}\right]^2}{\left\{\left[r_e+r_b\right]\left[1+\frac{R_L}{r_c+r_b}\right]+R_G\left[1+\frac{R_L+r_e}{r_c+r_b}\right]-\alpha\left[r_b+R_G\right]\right\}^2}$ IF $R_L+r_e \ll r_c$, $r_e+r_b \ll r_c$ $\cong \frac{4R_GR_L\alpha^2}{\left[r_e+(1-\alpha)(r_b+R_G)\right]^2}$	$= \frac{4R_G}{R_L\left\{\left[1+\frac{r_e}{R_L}\right]\left[1+\frac{r_b+R_G}{r_c}\right]+\left[1-\alpha\right]\left[\frac{r_b+R_G}{R_L}\right]\left[1+\frac{r_b}{r_c}\right]\right\}^2}$ IF $r_e \ll R_L$, $r_b+R_G \ll r_c$ $\cong \frac{4R_G}{R_L\left[1+\left(\frac{r_b+R_G}{R_L}\right)(1-\alpha)\right]^2}$
INSERTION GAIN = POWER TO LOAD / POWER GEN. WOULD DELIVER TO SAME LOAD	$= \left[1+\frac{R_G}{R_L}\right]^2\frac{\alpha^2R_L^2}{\left\{\left[r_e+r_b+R_G\right]\left[1+\frac{R_L}{r_c+r_b}\right]-\alpha r_b\right\}^2}$ IF $R_L \ll r_c$ $\cong \left[1+\frac{R_G}{R_L}\right]^2\frac{\alpha^2R_L^2}{\left[r_e+R_G+r_b(1-\alpha)\right]^2}$	$= \frac{\left[1+\frac{R_G}{R_L}\right]^2\left[\alpha-\frac{r_e+r_b}{r_c+r_b}\right]^2}{\left\{\left[\frac{r_e+r_b}{R_L}\right]\left[1+\frac{R_L}{r_c+r_b}\right]+\frac{R_G}{R_L}\left[1+\frac{R_L+r_e}{r_c+r_b}\right]-\alpha\left[\frac{r_b+R_G}{R_L}\right]\right\}^2}$ IF $R_L+r_e \ll r_c$, $r_e+r_b \ll r_c$ $\cong \left[1+\frac{R_G}{R_L}\right]^2\left[\frac{\alpha R_L}{r_e+(1-\alpha)(r_b+R_G)}\right]^2$	$= \frac{\left[1+\frac{R_G}{R_L}\right]^2}{\left\{\left[1+\frac{r_e}{R_L}\right]\left[1+\frac{r_b+R_G}{r_c}\right]+\left[\frac{r_b+R_G}{R_L}\right]\left[1-\alpha\right]\left[1+\frac{r_b}{r_c}\right]\right\}^2}$ IF $r_e \ll R_L$, $r_b+R_G \ll r_c$ $\cong \frac{\left[R_L+R_G\right]^2}{\left[R_L+(1-\alpha)(r_b+R_G)\right]^2}$
MAXIMUM AVAILABLE GAIN	$= \frac{\alpha^2(r_c+r_b)}{(r_e+r_b)}\times\frac{1}{(1+\beta_b)^2}$ $\beta_b = \sqrt{\frac{r_e+(1-\alpha)r_b}{r_e+r_b}}$ IF $r_b \ll r_c$ $= \frac{\alpha^2r_c}{(r_e+r_b)(1+\beta_b)^2}$	$= \frac{\left[\frac{r_e+r_b}{r_c+r_b}-\alpha\right]^2}{\left[\frac{r_e+r_b}{r_c+r_b}\right]\left[\frac{r_e}{r_c+r_b}+(1-\alpha)\right]\left[1+\beta_e\right]^2}$ $\beta_e = \sqrt{\frac{\left[r_c+r_b\right]\left[r_e+(1-\alpha)r_b\right]}{\left[r_e+r_b\right]\left[r_e+(1-\alpha)(r_c+r_b)\right]}}$ IF $r_e+r_b \ll r_c$, $r_b \ll r_c$ $\cong \frac{r_c}{r_e+r_b}\times\frac{\alpha^2}{1-\alpha}\times\frac{1}{(1+\beta_e)^2}$ $\beta_e \cong \sqrt{\frac{r_c\left[r_e+(1-\alpha)r_b\right]}{\left[r_e+r_b\right]\left[r_e+(1-\alpha)r_c\right]}}$	$= \frac{1}{\left[\frac{r_b+r_c}{r_c}\right]\left[\frac{r_e}{r_c}+\frac{(1-\alpha)(r_c+r_b)}{r_c}\right]\left[1+\beta_c\right]^2}$ $\beta_c = \sqrt{\frac{r_e+(1-\alpha)r_b}{r_e+(1-\alpha)(r_c+r_b)}}$ IF $r_b \ll r_c$, $r_e \ll r_c$ $\cong \frac{1}{(1-\alpha)(1+\beta_c)^2}$ $\beta_c \cong \sqrt{\frac{r_e+(1-\alpha)r_b}{r_e+(1-\alpha)r_c}}$
LOAD AND GEN RES. FOR MAX AVAIL GAIN — R_{GM}	$= (r_e+r_b)\beta_b$	$= (r_e+r_b)\beta_e$	$= (r_c+r_b)\beta_c$ IF $r_b \ll r_c$ $\cong r_c\beta_c$
LOAD AND GEN RES. FOR MAX AVAIL GAIN — R_{LM}	$= (r_c+r_b)\beta_b$ IF $r_b \ll r_c$ $\cong r_c\beta_b$	$= \left[r_e+(1-\alpha)(r_c+r_b)\right]\beta_e$ IF $r_b \ll r_c$ $\cong \left[r_e+(1-\alpha)r_c\right]\beta_e$	$= \left[r_e+(1-\alpha)(r_c+r_b)\right]\beta_c$ IF $r_b \ll r_c$ $\cong \left[r_e+(1-\alpha)r_c\right]\beta_c$