

## Small-Signal

### Part VIII

By **ABRAHAM COBLENZ** and **HARRY L. OWENS**

Signal Corps Engineering Laboratories  
Fort Monmouth, New Jersey

**S**MALL-SIGNAL parameters are of considerable importance in specifying transistor operation. Most of the parameters discussed thus far in this series are based on small-signal operation, and large errors can result from neglect of this qualifying phrase.

#### Power Gain

In Part VII of this series, equations were developed which express the input and output resistance and the voltage gain of point-contact and junction transistors. Typical numerical values were given to serve as a general orientation with regard to orders of magnitude (see Table I). The power gain of an electrical device is, obviously, an important concept to the design engineer and equations will be developed for computation of this parameter of a transistor.

Figure 1 shows the familiar circuit of a generator with voltage  $e_g$  and internal resistance  $R_g$  supplying a load whose resistance is

$R_L$ . The maximum power that can be developed across  $R_L$  for a given generator may be found as follows: Since

$$E = \frac{e_g R_L}{R_g + R_L} \quad (1)$$

$$P_o = \frac{e_g^2 R_L}{(R_g + R_L)^2} \quad (2)$$

Differentiation with respect to  $R_L$  shows that the power delivered across the load will be a maximum if  $R_g$  is equal to  $R_L$ . Under these conditions the maximum possible power that can be drawn from generator  $e_g$  under any circumstances will be

$$P_m = \frac{e_g^2}{4R_g} \quad (3)$$

The power output of a grounded-base transistor amplifier (Fig. 2) is in general given by  $i_2^2 R_L$  where  $i_2$  is

$$i_2 = \frac{-e_g(r_b + r_m)}{[(R_g + r_b + r_o)(R_L + r_b + r_c) - r_b(r_b + r_m)]} \quad (4)$$

The power gain of the transistor will then be given by the ratio of  $i_2^2 R_L$  to  $P_m$ , the maximum power which may possibly be drawn from the generator (generator internal resistance exactly matched by input resistance of the transistor). In general, this will not be the case and less power will be drawn from the generator than is indicated by Eq. 3. The power gain is given by

$$PG = \frac{i_2^2 R_L}{e_g^2 / 4R_g} = \frac{4R_L R_g i_2^2}{e_g^2} \quad (5)$$

The power gain so determined for a given value of  $i_2$  and  $R_L$  will be a lower limit or a minimum value since the power expression in the denominator is the maximum pos-

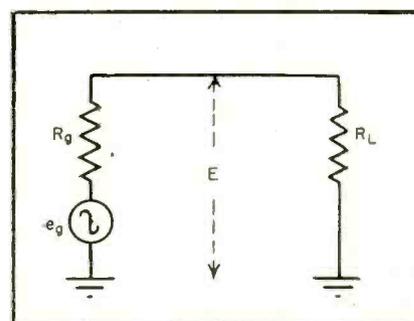


FIG. 1—Equivalent circuit illustrates condition for maximum load power

sible under these conditions.

Substituting the value of  $i_2$  from Eq. 4 into the expression for the power gain in Eq. 5

$$PG = \frac{4R_L R_g (r_b + r_m)^2}{[(R_g + r_b + r_o)(R_L + r_b + r_c) - r_b(r_b + r_m)]^2} \quad (6)$$

This expression can be simplified somewhat since  $r_o$  is small compared to  $r_o$  or  $r_m$  (see Table I) and Eq. 6 becomes

$$PG = \frac{4R_L R_g r_m^2}{[(R_g + r_b + r_o)(R_L + r_c) - r_b r_m]^2} \quad (7)$$

#### Typical Values

Using the typical values of the parameters as given in Table I for the point-contact transistor, a typical power gain is very nearly 100 representing 20 db whereas for the junction type a typical power gain is very nearly 440 representing 46 db.

#### Importance of Alpha

It is often desirable to know power gain as a function of the alpha of the circuit. Dividing numerator and denominator of Eq. 6 by  $(r_o + r_b)^2$

#### INDEX TO PREVIOUS ARTICLES IN THIS SERIES

Part I, p 98, March 1953.

Part II, p 138, April 1953.

Part III, p 162, May 1953.

Part IV, p 164, June 1953.

Part V, p 158, July 1953.

Part VI, p 156, August 1953.

Part VII, p 156, September 1953.

# Transistor Operation

Different approaches to mathematical analysis of small-signal transistor operation are explained in detail, along with listings of advantages and disadvantages of each. Power gain and other important parameters are discussed.

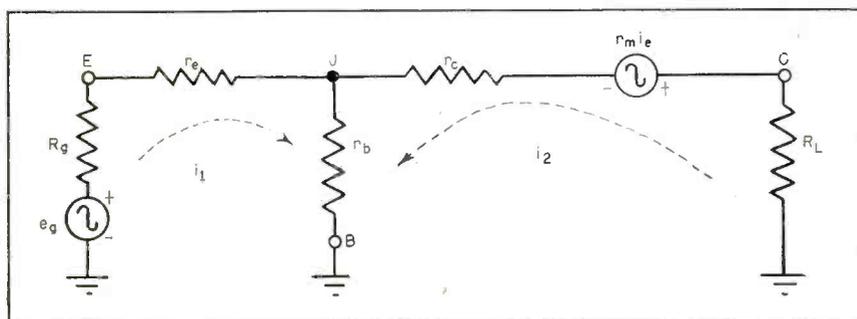


FIG. 2—Equivalent circuit of grounded-base transistor from which design equations for small-signal operation are devised

$$PG = \frac{4R_L R_g \alpha^2}{\left[ (R_g + r_b + r_e) \left( 1 + \frac{R_L}{r_e + r_b} \right) - r_b \alpha \right]^2} \quad (8)$$

Similarly, the power gain may be expressed as a function of  $VG$  and  $\alpha$ .

$$PG = \frac{4R_g \alpha (VG)}{(R_g + r_e + r_b) \left( 1 + \frac{R_L}{r_b + r_e} \right) - r_b \alpha} \quad (9)$$

Also, the expression for the  $VG$  obtained in the previous article<sup>1</sup> may be written as a function of  $\alpha$ .

$$VG = \frac{\alpha R_L}{(R_g + r_e + r_b) \left( \frac{R_L}{r_b + r_e} + 1 \right) - r_b \alpha} \quad (10)$$

Equations 8, 9 and 10 show why alpha is one of the important parameters for evaluation and comparison of transistors. Also, the relation  $\alpha = -i_c/i_e$  shows the further usefulness of alpha as a comparison number for current gain.

## Grounded-Base Equations

In the previous article of this series<sup>1</sup>, Eq. 11 and 12 were derived

to express conditions in the grounded-base transistor arrangement.

$$i_1(R_g + r_e + r_b) + i_2 r_b - e_g = 0 \quad (11)$$

$$i_1(r_b + r_m) + i_2(R_L + r_b + r_c) = 0 \quad (12)$$

If in Eq. 11  $r_e + r_b = r_{11}$  and  $r_{12} = r_b$  according to the definitions laid down in the previous article<sup>1</sup>, and  $v_1 = e_g - i_1 R_g$ , which is the net voltage acting at the transistor terminals, Eq. 11 becomes

$$v_1 = r_{11} i_1 + r_{12} i_2 \quad (13)$$

In deriving Eq. 12 it was assumed that there was no generator voltage acting in loop 2 of Fig. 2. If there were, Eq. 12 could be written as

$$r_{21} i_1 + i_2 R_L + i_2 r_{22} = e_{g2} \quad (14)$$

where again  $r_m + r_b = r_{21}$  and  $r_e + r_b = r_{22}$ . As before, if  $v_2 = e_{g2} - i_2 R_L$ , the net effective voltage after the  $i_2 R_L$  drop in Eq. 14 becomes

$$v_2 = r_{21} i_1 + r_{22} i_2 \quad (15)$$

## Small-Signal Parameters

Consider a dependent variable  $y$  which varies with, or is a function of an independent variable  $x$ . Mathematically

$$y = y(x) \quad (16)$$

Differentiating both sides with respect to  $x$

$$\frac{dy}{dy} = \frac{dy(x)}{dx} \quad (17)$$

Essentially, Eq. 17 is a tautology, or self-evident identity, and does not provide any particularly useful information. But in the form

$$dy = \frac{dy(x)}{dx} dx = y' dx \quad (18)$$

the equation states a very important and useful fact. Equation 16 will, in general, avoiding special curves, give a graph like in Fig. 3.

Equation 18 says that if, for a given or selected change in  $x$ , say of  $dx$ , it is desired to find the corresponding change in the value of  $y$ , when  $y$  depends on  $x$  in the manner shown in Fig. 3 and symbolized in mathematics shorthand by Eq. 16, the slope of the curve at the point in question is multiplied by the given value of  $dx$ .

If  $y$  is a dependent variable which varies with either or both of two parameters,  $x$  and  $z$ , Eq. 16

Table I—Typical Transistor Parameters

Parameter	Point Contact	Junction
$r_e$	150 ohms	25 ohms
$r_b$	120 ohms	500 ohms
$r_m$	35 kilohms	0.96 megohm
$r_c$	15 kilohms	1.0 megohm
$R_g$	500 ohms	500 ohms
$R_L$	20 kilohms	100 kilohms
$\alpha$	2.3	0.96
$e_g$	0.01 volt	0.001 volt

will then become

$$y = y(x, z) \quad (19)$$

Since a change in  $y$ ,  $dy$ , may now be due to a change in either or both of  $x$  and  $z$ , an equation entirely analogous to Eq. 17 would be Eq. 20

$$dy = \left. \frac{dy}{dx} \right]_{z=k} dx + \left. \frac{dy}{dz} \right]_{x=k} dz \quad (20)$$

This is usually written

$$dy = \frac{\partial y}{\partial x} dx + \frac{\partial y}{\partial z} dz$$

The dimension of each term on the right-hand side of Eq. 20 is  $dy$ , since  $dx$  and  $dz$  cancel—at least dimensionally. This must be so for any physically valid interpretation. Also  $dy/dx$ , the first term right-hand side, is not quite the same as in Eq. 17 because the change in  $dy$  is measured for a unit change in  $dx$ , while  $z$  is kept constant. A similar remark applies to  $dy/dz$  for  $x = \text{constant}$ .

In general,  $V_e$ , the d-c emitter voltage, will depend on the emitter and collector currents or on the operating point. Thus

$$V_e = V_e(I_e, I_c) \quad (21)$$

Applied to Eq. 19,  $y \approx V_e$ ,  $I_e \approx x$ ,  $I_c \approx z$ . The dependent variable is voltage  $V_e$  and the independent variables are  $I_e$  and  $I_c$ . This shows that the transistor is a current-operated device and the currents are the independent variables. In transistor work the currents are adjusted to the correct operating point, and the voltages appearing are then fixed by the current values.

To illustrate further the effect of this concept in practice, the static characteristics of transistors are plotted, by common consent, with current as abscissa and voltage as ordinate, in keeping with the mathematical convention of plotting the independent variable along the  $x$  direction.

Analogous to Eq. 20, obtained from 19, is Eq. 22 for Eq. 21.

$$dV_e = \left. \frac{dV_e}{dI_e} \right]_{I_c=k} dI_e + \left. \frac{dV_e}{dI_c} \right]_{I_e=k} dI_c \quad (22)$$

The d-c collector voltage also depends on emitter and collector currents, and

$$V_c = V_c(I_e, I_c) \quad (23)$$

Again on differentiating

$$dV_c = \left. \frac{dV_c}{dI_e} \right]_{I_c=k} dI_e + \left. \frac{dV_c}{dI_c} \right]_{I_e=k} dI_c \quad (24)$$

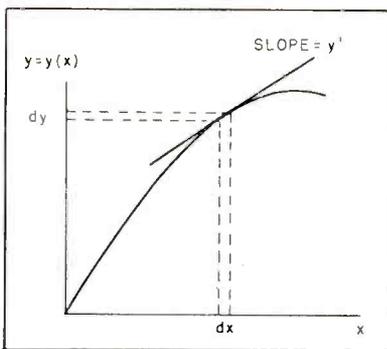


FIG. 3—Typical curve showing  $y$  as a function of  $x$ . Slope is  $y' = dy(x)/dx$

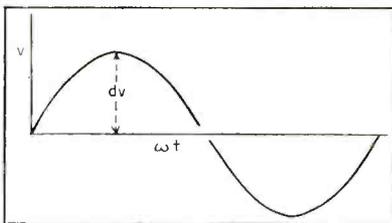


FIG. 4—Sinusoidal voltage (or current) used to simulate differential increments

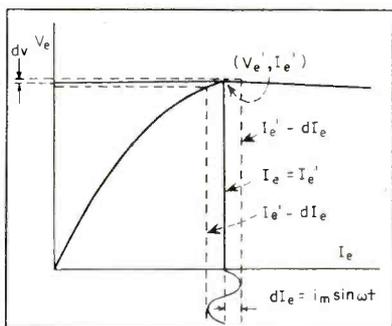


FIG. 5—Plot of emitter voltage versus emitter current

Equations 22 and 24 are extremely important in transistor analysis.

### Small Changes

A simple explanation of Eq. 24 might be based on an a-c voltage and current wave as shown in Fig. 4, where the sinusoidal voltage wave  $v = v_m \sin \omega t$  where  $v_m = dv$ .

In an interval  $\omega t = \pi/2$ , the amplitude changes from 0 to  $dv$ , and in the next interval of time from  $dv$  to 0, so that, disregarding the sign, the amplitude changes by  $dv$  for each  $\frac{1}{2}$  cycle. In general, Eq. 22 and 24 apply properly if  $dv$  is actually infinitesimal, or extremely small, but in actual application a practical value of  $dv$  may be used.

In Fig. 5 is shown a possible plot of d-c values of voltage vs cur-

rent, say of the emitter voltage  $V_e$  and emitter current  $I_e$  for a transistor. Assuming the operating point of the transistor is at  $V_e', I_e'$ , if the emitter current is modulated sinusoidally by a small signal so that  $I_e$  varies at most by  $dI_e$ , then  $I_e$  will vary from  $I_e'$  to  $I_e' + dI_e$ , from  $I_e' + dI_e$  to  $I_e'$ , from  $I_e'$  to  $I_e' - dI_e$ , and so forth. The quantitative relation between the change  $dI_e$  in  $I_e'$  and the change  $dV_e$  in  $V_e$ , assuming that  $V_e$  depends on  $I_e$  alone, is

$$dV_e = \frac{dV_e}{dI_e} dI_e$$

as in Eq. 18.

Just as  $dy/dx = y'$  is the slope of the  $y = y(x)$  curve, so  $dV_e/dI_e$  is the slope of the characteristic curve of Fig. 5. (Note that  $dV_e/dI_e$  has the dimensions of a resistance, so that in this case the slope is a resistance, hence designated by  $R$ .) The slope of the curve must be taken virtually at a point if the relation is to be true.

$$dV_e = R dI_e \quad (25)$$

### Signal Limitations

If in using Eq. 25 the slope over a large region is used the condition shown in Fig. 6 occurs. The slope at point  $(V_e', I_e')$  is actually  $S_1$  as shown, but using a large swing about  $(V_e', I_e')$  gives a slope  $S_2$  which is in general not equal to  $S_1$ . Thus, using a large distance about the point in question to compute the slope may introduce large errors; the larger the distance used, the larger may be the error, particularly as the curve departs more and more from a straight line. If a sinusoidal current is used to simulate  $dI_e$ , it must be a very small current or signal, or else large errors may be introduced.

In view of these remarks, the differential quantity  $dV_e$  is representable, for purposes of a laboratory experiment, by an a-c voltage, and  $dI_e$  by an a-c current, provided they are suitably small.

Regardless of whether  $V_e$  is a function of one or two variables, the remarks about the size of the signal are still valid.

Equations 22 and 24 may be written (Eq. 26 and 27 respectively) in terms of small a-c  $v$ 's and  $i$ 's by applying the foregoing principles.

$$v_e = \left. \frac{v_e}{i_e} \right]_{I_c=k} i_c + \left. \frac{v_e}{i_c} \right]_{I_e=k} i_c \quad (26)$$

$$v_c = \left. \frac{v_c}{i_e} \right]_{I_c=k} i_e + \left. \frac{v_c}{i_c} \right]_{I_e=k} i_c \quad (27)$$

There are several comments which must be made about these two equations: (1) Note the use of small letters for a-c values, and capitals for d-c values, in keeping with the convention adopted by the authors for this series, and which is a proposed standard.

(2) Consider the  $I_c = k$ ,  $I_e = k$  factors of Eq. 26 and 27. If a change  $dI$  is representable by a small a-c current,  $i \sin \omega t$ , as has been shown, of amplitude  $i$ , then 0 change in d-c current,  $I_c = k$ , or  $I_e = k$ , will be represented by an a-c current of 0 amplitude. Using this fact we may properly write Eq. 26 and 27 as

$$v_e = \left. \frac{v_e}{i_e} \right]_{i_e=0} i_c + \left. \frac{v_e}{i_c} \right]_{i_e=0} i_c \quad (28)$$

$$v_c = \left. \frac{v_c}{i_e} \right]_{i_e=0} i_e + \left. \frac{v_c}{i_c} \right]_{i_e=0} i_c \quad (29)$$

(3) At  $i_e = 0$ ,  $v_e/i_e$  is  $r_{11}$  as described in Part VII of this series.

$$r_{11} = \left. \frac{v_e}{i_e} \right]_{i_e=0} \quad (30)$$

Using the definitions laid down in Part VII

$$\left. \frac{v_e}{i_c} \right]_{i_e=0} = r_{12}$$

$$\left. \frac{v_c}{i_e} \right]_{i_e=0} = r_{21}$$

$$\left. \frac{v_c}{i_c} \right]_{i_e=0} = r_{22}$$

Hence Eq. 28 and 29 can be rewritten

$$v_e = r_{11}i_e + r_{12}i_c \quad (31)$$

$$v_c = r_{21}i_e + r_{22}i_c \quad (32)$$

Equation 31 states that a potential  $v_e$  is acting in a circuit and produces the two potential drops  $r_{11}i_e$  and  $r_{12}i_c$  which together make up  $v_e$ . Similar remarks apply to Eq. 32. Replacing subscript  $e$  in the emitter circuit by subscript 1, and subscript  $c$  by 2, Eq. 31 and 32 become

$$v_1 = r_{11}i_1 + r_{12}i_2 \quad (33)$$

$$v_2 = r_{21}i_1 + r_{22}i_2 \quad (34)$$

Equations 13 and 15 are identical to Eq. 33 and 34 respectively, but were arrived at by entirely different methods of reasoning. Equations 13 and 15 were obtained from a circuit analysis using Kirchhoff's law for the voltage loops; Eq. 33 and 34 were derived on a mathematical basis from the simple concept that  $V_e$  and  $V_c$  are dependent on both  $I_e$  and  $I_c$  in small-signal operation.

As has been mentioned, the ultimate fact mathematically expressed by these equations is that, given all the other data, it is possible to find two numbers,  $i_1$  and  $i_2$ , that satisfy these equations. The fact that they happen to represent a current, in milliamperes perhaps, does not influence the mathematical description of how to determine these two numbers. Since Eq. 33 and 34 are linear equations (that is, they contain no products like  $i_1 \times i_2$ , or  $i$  raised to some power) the delineation is unique; that is, the equations will define only one pair of values,  $i_1$  and  $i_2$ , and no other.

Consequently, whatever assumptions were made to find Eq. 33 and 34 are also binding on Eq. 13 and

15 because they represent the same numbers.

### Assumptions

First, from the discussion in connection with Eq. 31 and 32, the parameters are open-circuit values, that is,  $i_c = 0$ , when measuring  $r_{11}$  and  $r_{21}$  and  $i_e = 0$  when measuring  $r_{12}$  and  $r_{22}$ . This condition has already been met for Eq. 13 and 15'.

Second, from the discussion in connection with Eq. 26 and 27, the parameters are measured using ratios of a-c values. This condition has also been met'.

Third, the symbol convention that

$$v_1 = e_{o1} - i_1 R_{\theta}$$

$$v_2 = e_{o2} - i_2 R_L$$

assumed to obtain Eq. 13 and 15, is binding on Eq. 33 and 34.

Last, and probably most important, from the discussion in connection with Eq. 26 and 27, the use of small signals is assumed. This means that the four-pole parameters  $r_{11}$ ,  $r_{12}$ ,  $r_{21}$  and  $r_{22}$  are small-signal parameters.

In selecting an amplitude for the measurement of the four-pole parameters, several factors must be considered.

If the signal is not truly small, different signal amplitudes will give different values of parameters.

In practice, a signal of some convenient size is selected, and the parameter, say  $r_{11}$ , measured. Then the signal is reduced slightly, and the parameter measured again. If the value of  $r_{11}$  obtained in the second trial is close to the  $r_{11}$  originally obtained, within the accuracy desired, the signal is a small signal and conversely.

It might appear that there is no objection to using a very small amplitude signal originally, so that there is no question about the signal being small. If the  $V$ - $I$  characteristic is very curved, it is very difficult to guess what constitutes a truly small signal. The test suggested above should therefore always be made. Too small a signal introduces measuring problems such as noise and the general difficulty of measuring fractions of a microvolt at a-c.

When feasible, the parameter may be measured by the volt-ammeter method tacitly assumed here,

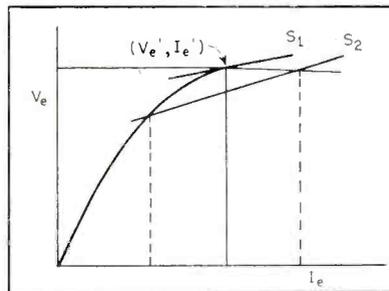


FIG. 6—Sketch shows error in determining slope with large signal input

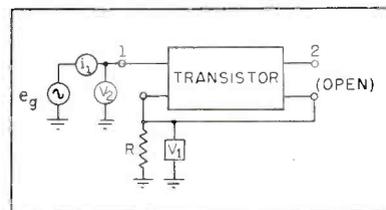


FIG. 7—Connections for measuring  $r_{11}$

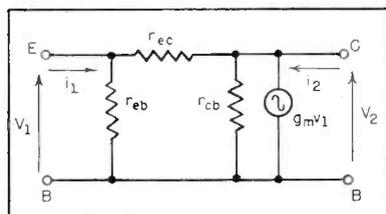


FIG. 8—Equivalent pi circuit of transistor

and the resulting value compared to that obtained by using an entirely different measuring scheme. Other possible methods include the bridge method, scope presentation, and variations and combinations of these. The results must compare within the accuracy desired if the signal is truly small.

### Open Circuit

Let us now review in a general way the data that has been assembled regarding  $r_{11}$ ,  $r_{12}$ ,  $r_{21}$  and  $r_{22}$ . Because these parameters are so important in the specification of transistor characteristics, much experimental and theoretical work has been done and is continuing regarding these and other suitable parameters.

These parameters are called small-signal open-circuit grounded-base four-pole parameters. In this and previous articles the terms small-signal, grounded-base, and four-pole parameters have been explained. To understand the open-circuit term, consider Fig. 7 which shows the circuit arrangement for measurement of  $r_{11}$ .

Both  $V_1$  and  $V_2$  are vacuum-tube voltmeters, and  $R$  is a small resistance of known value placed in the signal generator return to measure the current  $i_1 = V_1/R$ . Voltmeter  $V_2$  measures the voltage across the input of the transistor, and  $r_{11} = V_2/i_1$ . There is not a true open circuit across the output or collector circuit since whatever scheme is used to bias the collector, a d-c return path to the base is essential, and thus a d-c path is always present to act as a closed circuit across the collector. The internal resistance of the d-c bias supply may be made very high by using a choke of several hundred henries but not infinite. In most tests  $f = 270$  cps.

Also, there is an a-c shunting path always present due to internal transistor and stray capacitance. The impedance of this shunting path decreases as the frequency increases.

A typical value of  $C_c$ , the collector capacitance, is  $50 \mu\text{f}$  and a typical circuit may have an additional stray capacitance of  $10 \mu\text{f}$ . At 270 cps,  $X_c = 9.8$  megohms. If  $r_{22}$  were three megohms, at 270 cps

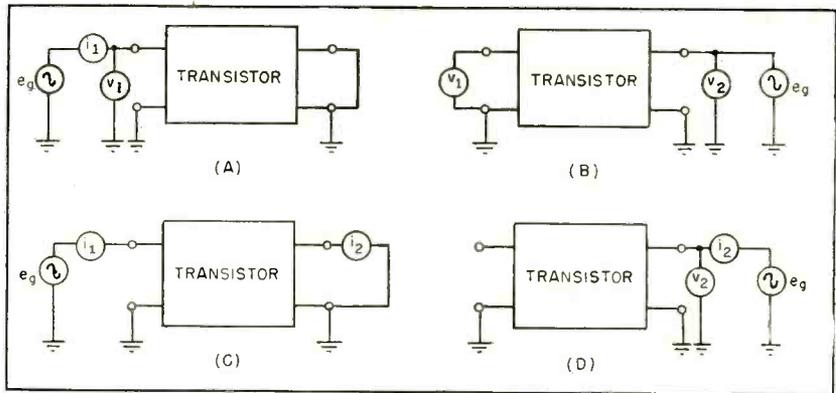


FIG. 9—Sketches show connections for measuring  $g$ 's or short-circuit conductances

the 9.8 megohms would represent a shunt path of some importance, and it is not true that measurements are being made under open-circuit conditions.

Two principal approaches to this problem have been adopted.

(1) The arbitrary convention that the circuit is open if the shunt resistance is fifty times the internal transistor resistance. Thus when measuring  $r_{11}$  and  $r_{21}$ , where the collector circuit must be open, the shunt path shall be  $50 \times r_{22}$ ; and when measuring  $r_{12}$  or  $r_{22}$  the shunt path shall be  $50 \times r_{11}$ . Or, where feasible, the collector-to-base resistance (or emitter-to-base resistance) may be doubled. If the value of the parameter measured remains invariant within the accuracy desired, the circuit is truly an open circuit.

(2) Measurement of other parameters such as the  $g$ 's and  $h$ 's described below to characterize transistor operation, whose measurement does not require the use of an open circuit across high impedance elements. Then, if desired, parameters such as the  $r$ 's may be derived mathematically from the quantities measured. Or, the performance of the transistor may be expressed in terms of these new parameters directly, it being assumed that with experience engineers will readily compare transistor performance in terms of these parameters.

### Conductances

It was mentioned that the equivalent T as characterized by Eq. 13 and 15 is not the only equivalent circuit for the specification of the

allegorical black box. The black box or transistor is equally well represented by an equivalent circuit which is a  $\pi$  network,<sup>2</sup> as in Fig. 8. This circuit is analyzed by means of Eq. 35 and 36 using conductances.

$$i_1 = g_{11}v_1 + g_{12}v_2 \quad (35)$$

$$i_2 = g_{21}v_1 + g_{22}v_2 \quad (36)$$

Using Eq. 35 and 36 all the four  $g$ 's can be measured. For illustration, setting  $v_2 = 0$ ,  $g_{11} = i_1/v_1$  and similarly one can find  $g_{12}$ ,  $g_{21}$  and  $g_{22}$ ; see Fig. 9. These  $g$ 's are sometimes called the short-circuit conductances.

While it is true that resistances and conductances are reciprocals, the  $r_{11}$  which appears in Eq. 13 is not the reciprocal of  $g_{11}$ . In Eq. 13  $r_{11}$  was obtained using an open-circuited collector, but  $g_{11}$  is obtained by using a short-circuited collector. Hence,  $r_{11}$  is not  $1/g_{11}$  and is, in fact, given by

$$r_{11} = \frac{g_{22}}{g_{11}g_{22} - g_{12}g_{21}} \quad (37)$$

To obtain  $g_{11}$ ,  $v_2$  is set to 0. In other words, the output is short circuited. This does away with the need for establishing an open circuit. Like any other good idea in engineering, it has its drawbacks. To find  $g_{12}$  the emitter circuit may be shorted, making  $v_1 = 0$ , and so on for all the other parameters.

The disadvantage of the  $g$  method is that some point-contact transistors may be short-circuit unstable. If the input or the output is a-c short-circuited, the units may break into parasitic oscillations. Fortunately, the applicability of this method is greatest for junction types, where this instabil-

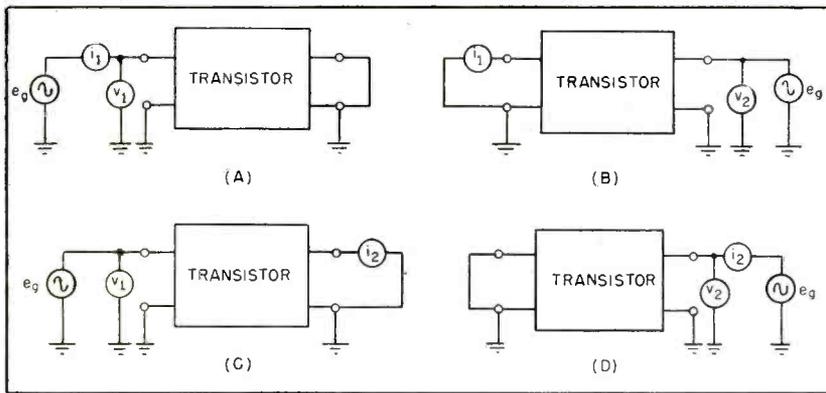


FIG. 10—Circuits for measuring  $h$ 's or hybrid transistor parameters

ity is not normally encountered. The inference is that for point-contact units, the  $r$ 's may be suitable, and with junction units, the  $g$ 's may be preferable.

### Hybrid Parameters

Because the use of the  $g$ 's suggests that two different parameters are needed to take care of point-contact and junction units, an entirely different set has been suggested.<sup>3</sup> These are the hybrid parameters, or  $h$ 's, characterizing the transistor by the following equations

$$v_1 = h_{11}i_1 + h_{12}v_2 \quad (38)$$

$$i_2 = h_{21}i_1 + h_{22}v_2 \quad (39)$$

To find  $h_{11}$ , the output is shorted, (see Fig. 10)  $v_2 = 0$ , and

$$h_{11} = \frac{v_1}{i_1} \quad (40)$$

The  $r_{11}$  of Eq. 13 was obtained with open-circuited output, hence  $h_{11}$  is not the same as  $r_{11}$ . As  $g_{11}$  was obtained with  $v_2 = 0$ ,  $h_{11} = 1/g_{11}$ , so that the  $g_{11}$  of Eq. 35 is the reciprocal of  $h_{11}$  in Eq. 35; and since the reciprocal of a conductance is a resistance,  $h_{11}$  is of the nature of a resistance, dimensionally.

To find  $h_{12}$ ,  $i_1$  is made 0 by making the input an open circuit, placing the generator in the collector circuit, and measuring  $v_1$  and  $v_2$ . Then

$$h_{12} = \frac{v_1}{v_2} \quad (41)$$

As this is the ratio of two voltages,  $h_{12}$  has no dimensions. A similar analysis will show, for Eq. 39, that  $h_{21} = i_2/i_1$  for a short-circuited

collector. This is  $\alpha_{ce}$  by definition;  $h_{22} = i_2/v_2$  is conductance but not identical with  $g_{22}$  of Eq. 36 since all the parameters of Eq. 36 are obtained under short-circuit conditions. Note that  $h_{22}$  is obtained under open-circuit conditions.

Thus the hybrid parameters contain two pure numerics,  $h_{12}$  and  $h_{21}$ , a resistance  $h_{11}$  and a conductance  $h_{22}$ . They possess, however, some of the advantages of both the  $r$ 's and the  $g$ 's:

(1) In the matter of avoiding the necessity for maintaining an open circuit in the high-resistance collector, the  $h$  method shares the advantage of the  $g$  system in that  $h_{11}$  and  $h_{21}$  are made with collector shorted.

In the matter of avoiding the necessity for maintaining a short circuit in the low-resistance emitter circuit, the  $h$  system shares the advantage of the  $r$  system, since both  $h_{12}$  and  $h_{22}$  are made with emitter open circuited. Since the input resistance of some transistors may be quite low, of the order of tens of ohms, difficulties are encountered in effectively a-c short circuiting such impedances. In the  $h$  system, the input is open circuited for such measurements ( $h_{12}$  and  $h_{22}$ ).

(2) Measurements for alpha in the  $g$  and  $r$  methods are indirect. In the  $r$  method,  $r_{21}$  and  $r_{22}$  are found; their ratio is alpha =  $r_{21}/r_{22}$ . In the  $g$  method, alpha =  $g_{21}/g_{22}$ ; but in the hybrid parameter method, alpha =  $-h_{21}$  directly.

The principal disadvantages of the  $h$ 's is that they are not directly amenable to circuit analysis. Whereas most engineers are acquainted with resistances and con-

ductances and use them readily in circuit analysis, few are prepared to use  $h$ 's directly. If transformations are necessary, and these are quite cumbersome, the advantages mentioned above may well be overshadowed. Engineers are currently studying the relative merits of each set of parameters. The indications are that considerable additional experience is needed before a single set of parameters to characterize transistors will be generally adopted.

### Resume

In resume, the following are the salient points:

(1) Typical power gains that are possible with the grounded-base transistor connection are: point-contact, 20 db; junction, 46 db.

(2) In transistor terminology  $r_{11}$ ,  $r_{12}$ ,  $r_{21}$  and  $r_{22}$  are called small-signal, grounded-base, open-circuit, four-pole, equivalent-circuit parameters. Each part of the name has significance.

(3) A test to determine whether a signal is truly a small signal is to measure the parameters using a selected amplitude of signal, decrease the signal by approximately 50 percent and remeasure the parameters. The measured values must compare within the orders of accuracy required.

(4) Either the  $r$ 's, or the  $g$ 's or  $h$ 's may be used to characterize the transistor equivalent circuit. Considerations such as common usage, short-circuit stability, and effective open and short circuit are involved in selecting the parameters for a given analysis.

### REFERENCES

- (1) Abraham Coblenz and Harry L. Owens, Equivalent Transistor Circuits and Equations, TRANSISTORS, Theory and Application, Part VII, ELECTRONICS, p 156, Sep. 1953.
- (2) L. J. Giacoletto, Junction Transistor Equivalent Circuits and Vacuum tube Analogy, Proc IRE, p 1490, v 40 11, Nov. 1952.
- (3) D. A. Alsberg of Bell Telephone Laboratories, Murray Hill, N. J., before IRE, March, 1953, in talk entitled "Transistor Metrology."
- (4) R. M. Ryder and R. J. Kircher, Some Circuit Aspects of the Transistor, BSTJ, p 367, 28, 3 Jul. 1949.
- (5) Measurement of transistor parameters, including the  $r$ 's,  $g$ 's and  $h$ 's, in Signal Corps Interim Basic Section on Transistors, entitled: "Transistors, Crystal Diodes, and Related Semiconductor Devices" MIL-T-12679 (Sig C). Requests from concerns and individuals actively engaged in semiconductor device activity should be addressed: Director, Evans Signal Laboratory, Belmar, N.J., Attn: Thermionics Branch.